



# Having enough and not having too much: A characterization of sufficientarianism–limitarianism

João V. Ferreira , Foivos Savva \*

Department of Economics, University of Southampton, UK

## ARTICLE INFO

### JEL classification:

D31  
D63

### Keywords:

Sufficientarianism  
Limitarianism  
Distributive justice  
Social welfare

## ABSTRACT

Sufficientarianism, a prominent framework in distributive justice, asserts that everyone should have enough resources to meet a minimum threshold. Limitarianism, by contrast, holds that no individual should possess more than a specified upper limit of income or wealth. While the latter has gained attention in political philosophy and policy debates, it remains largely unexplored in formal normative economics. This paper bridges this gap by offering an axiomatic characterization of a social welfare criterion that integrates sufficientarian and limitarian principles. We formalize these dual commitments and investigate their implications for resource allocation. The analysis sheds light on the theoretical underpinnings of this hybrid approach and its potential relevance for normative analysis.

“To focus on inequality, which is not in itself objectionable, is to misconstrue the challenge we actually face. Our basic focus should be on reducing both poverty and excessive affluence”.

[Harry G. Frankfurt, On Inequality]

## 1. Introduction

*Sufficientarianism* is a prominent approach to distributive justice in political philosophy. As originally formulated by Frankfurt (1987), it posits that society should prioritize maximizing the number of people who “have enough” resources to surpass a specified threshold that defines sufficiency.<sup>1</sup> This approach embodies two core principles: a “positive thesis”, which asserts that everyone should have enough, and a “negative thesis”, which contends that once individuals have enough, no further distributional considerations are morally significant (Casal, 2007). In economics, a growing body of literature has explored axiomatic characterizations of various versions of sufficientarianism (e.g., Alcantud et al. 2022, Chambers and Ye 2024, Bossert et al. 2023).

*Limitarianism*, by contrast, holds that no individual should possess more than a specified upper threshold of income or wealth (Robeyns 2017, 2022, 2024). It is regarded as a “partial account of justice”, which should be combined with different theories of justice below the limitarian threshold (Robeyns 2022, p. 8). Limitarianism has attracted increasing attention in political philosophy (e.g., Timmer 2021, Huseby 2022) and public debates (e.g., CNN 2019, Bloomberg 2022, The

Guardian 2024, Strain 2024, Ferreira et al. 2024b). Proponents argue that limiting extreme wealth could address pressing societal needs and mitigate adverse externalities of wealth accumulation on social, democratic, and environmental systems. Recent empirical evidence also suggests that there is public support for policies that limit extreme levels of income and wealth (Ferreira et al. 2024a, Perez-Truglia and Yusof 2024). However, limitarianism remains virtually unexplored in the formal normative economics literature.

In this paper, we are the first to provide an axiomatic characterization of a criterion that incorporates sufficientarian and limitarian concerns. Our social welfare criterion assumes the existence of a lower sufficientarian threshold and an upper limitarian threshold such that, for any two given allocations  $x$  and  $y$ ,  $x$  is better than  $y$  if and only if the number of people within the two thresholds is greater in  $x$  than in  $y$ . We demonstrate that a single plausible axiom, *No Extremes*, is what distinguishes our hybrid criterion from headcount sufficientarianism. This axiom posits that there is an equal distribution of resources such that a move from a single individual to either the minimum or the maximum of the resource scale constitutes a social deterioration.

As with sufficientarianism, the plausibility of our criterion hinges on the interpretation of the thresholds and the “currency” of normative concern. Regarding the currency, we consider that a criterion that includes a limitarian threshold is most appropriate in the domain of resources such as income or wealth. Regarding the thresholds, there are at least two plausible interpretations. The first interpretation is based

\* Correspondence to: Building 58, Highfield Campus, SO17 1BJ, Southampton, UK.

E-mail address: [f.savva@soton.ac.uk](mailto:f.savva@soton.ac.uk) (F. Savva).

<sup>1</sup> See Roemer (2004), Casal (2007), and Huseby (2019) for reviews.

on *urgent needs and risks*. According to this view, the sufficientarian threshold marks a point below which income or wealth is unacceptable, such that improvements below this threshold cannot be regarded as progress (see Frankfurt 1987). A similar argument can be made for the limitarian threshold, where urgent risks related to extreme wealth concentration such as a threat to democracy render income or wealth above a certain threshold unacceptable (see Robeyns 2024). The second interpretation is based on a *quasi-egalitarian ideal*. Under this view, the focus of justice is on achieving a distribution where everyone has “roughly” the same level of resources. For instance, this could reflect a practical vision of a society where all individuals are part of the “middle class”.

Our paper contributes to two main strands of literature. First, it extends the burgeoning axiomatic literature on sufficientarianism (e.g., Alcantud et al. 2022, Bossert et al. 2022, 2023; Adler et al. 2023, Chambers and Ye 2024). In particular, we integrate a limitarian principle into existing characterizations of the *headcount* version of sufficientarianism, according to which what matters is the number of people who are above the sufficientarian threshold (Alcantud et al. 2022, Chambers and Ye 2024).<sup>2</sup> Second, our axiomatic characterization sheds new light into the building blocks of a plausible sufficientarian–limitarian approach to distributive justice. This complements existing discussions in the political philosophy literature on combining a limitarian threshold with other principles of justice (e.g., Robeyns 2017, 2022, 2024; Zwarthoed 2018, Nicklas 2021, Timmer 2021, Huseby 2022). Overall, this paper is a first step toward a canonical interpretation of justice principles that count both “having enough” and “not having too much” as essential evaluative criteria.

The rest of the paper is organized as follows. Next, we introduce our formal setting. In Section 3, we put forward our main axioms, and in Section 4 we demonstrate our main result. Section 5 concludes.

## 2. Setting

We consider a set  $N = \{1, \dots, n\}$  of individuals with  $n \geq 2$ . By  $x_i \in [0, 1]$  we denote  $i$ 's normalized level of resources like income or wealth, referred to as  $i$ 's bundle below. An allocation is an  $n$ -tuple  $\mathbf{x} = (x_1, \dots, x_n) \in [0, 1]^n \equiv \mathcal{X}$ . By convention, for any  $\mathbf{x}, \mathbf{y} \in \mathcal{X}$  and  $K \subseteq N$ , by  $(x_K, y_{-K}) = \mathbf{z}$  we denote the allocation where for all  $i \in K$ ,  $z_i = x_i$  and for all  $j \in N \setminus K$ ,  $z_j = y_j$ . Moreover, by slightly abusing notation, for any  $\mathbf{z} = ((x)_i, (y)_{-i})$ , we have  $z_i = x$  and for all  $j \neq i$ ,  $z_j = y$ . A permutation of  $\mathbf{x}$  is a bijection  $\sigma : N \rightarrow N$ .

A social welfare criterion is a complete, reflexive and transitive binary relation on  $\mathcal{X}$ , which we denote  $\succeq$ . As usual,  $>$  and  $\sim$  denote its asymmetric and symmetric parts, respectively.

**Definition 2.1.** A social welfare criterion  $\succeq^{SL}$  is *sufficientarian-limitarian* if there exists  $[\underline{u}, \bar{u}] \subseteq ]0, 1[$  such that, for all  $\mathbf{x}, \mathbf{y} \in \mathcal{X}$ , we have:  $\mathbf{x} \succeq \mathbf{y} \iff \#\{i \in N | x_i \in [\underline{u}, \bar{u}]\} \geq \#\{i \in N | y_i \in [\underline{u}, \bar{u}]\}$

In plain English, this social welfare criterion says that for any two allocations  $\mathbf{x}$  and  $\mathbf{y}$ ,  $\mathbf{x}$  is better than  $\mathbf{y}$  if and only if the number of people within the two thresholds is greater in  $\mathbf{x}$  than in  $\mathbf{y}$ . It incorporates sufficientarian concerns through the lower bound  $\underline{u}$  and limitarian concerns through the upper bound  $\bar{u}$ . To avoid the trivial case where  $[\underline{u}, \bar{u}] = [0, 1]$ , Definition 2.1 demands  $\underline{u}$  and  $\bar{u}$  to be elements of the open  $]0, 1[$  interval.

## 3. Conditions

In this section, we introduce our conditions. The first two are very standard in the social choice literature:

<sup>2</sup> For non-headcount criteria, see Bossert et al. (2022, 2023), (Adler et al., 2023), and Nakada and Sakamoto (2024). See Gravel et al. (2019) for an axiomatic analysis that combines *leximin* and *antileximax* principles.

**Definition 3.1.** A social welfare criterion  $\succeq$  satisfies *Separability (S)* if, for any  $\mathbf{x}, \mathbf{y}, \mathbf{x}', \mathbf{y}' \in \mathcal{X}$  and  $K \subseteq N$ , we have  $(x_K, y_{-K}) \succeq (x'_K, y'_{-K})$  implies  $(x_K, y'_{-K}) \succeq (x'_K, y'_{-K})$ .

**Definition 3.2.** A social welfare criterion  $\succeq$  satisfies *Anonymity (A)* if for all  $\mathbf{x}, \mathbf{y} \in \mathcal{X}$  where  $\mathbf{y}$  is a permutation of  $\mathbf{x}$ , we have  $\mathbf{x} \sim \mathbf{y}$ .

S dictates that the social welfare criterion should only restrict attention to the resources of the individuals whose bundles are at stake, and not to the “indifferent” ones. A is a minimal requirement of fairness in that the social welfare criterion should not give special consideration to any individual or group of individuals.

The following condition can be found in Chambers and Ye (2024) who use it to characterize a headcount sufficientarian social welfare criterion:

**Definition 3.3.** A social welfare criterion  $\succeq$  satisfies *Sufficientarian Judgment (SJ)* if for all  $i, j \in N$ ,  $i \neq j$ , for all  $x, y_i, z_j \in [0, 1]$  we have  $(x, \dots, x) > (y_i, (x)_{-i}) \Rightarrow (y_i, (x)_{-i}) \succeq (y_i, z_j, (x)_{-ij})$ .

The intuition behind SJ is that, if starting from an equal distribution of resources, a change in someone's bundle is a social deterioration, then, no matter how we change the bundle of any other individual, the allocation can never be strictly better than the original one. This is a key axiom of sufficientarianism that our criterion will also satisfy. In our setting, a change from an equal distribution that results in someone's deterioration implies that this person is either below the sufficientarian threshold or above the limitarian threshold. Therefore, given that the bundles of all other individuals lie between the two thresholds, and that equal negative weight is given to a move below the sufficientarian or above the limitarian threshold, such change can never be compensated by changing the bundle of any other individual.

Our next condition captures another essential aspect of the sufficientarian–limitarian criterion.

**Definition 3.4.** A social welfare criterion  $\succeq$  satisfies *No Extremes (NE)* if there exists  $\bar{\mathbf{x}} = (\bar{x}, \dots, \bar{x}) \in \mathcal{X}$  such that, for all  $i \in N$ ,  $\bar{\mathbf{x}} > (0, \bar{x}_{-i})$  and  $\bar{\mathbf{x}} > (1, \bar{x}_{-i})$ .

In plain English, this condition dictates the existence of an equal distribution of resources such that, starting from this equal distribution, if an individual's bundle changes to either the maximum or the minimum, then this is considered a social deterioration. No extremes encapsulates a minimal degree of egalitarianism and outlines the essence of our criterion: from a given equal distribution, neither too little nor too much is acceptable.

The following two conditions are more technical in nature. The first one ensures that the sufficientarian–limitarian set is closed. The second one ensures that it is an interval.

**Definition 3.5.** A social welfare criterion  $\succeq$  satisfies *Partial Upper Continuity (PUC)* if for all  $i \in N$  and  $x_i \in [0, 1]$ , the set  $\{y_i \in [0, 1] | \text{there exists } z_{-i} \in [0, 1]^{n-1} \text{ such that } (y_i, z_{-i}) \succeq (x_i, z_{-i})\}$  is closed.

**Definition 3.6.** A social welfare criterion  $\succeq$  satisfies *Partial Strict Convexity (PSC)* if for all  $i \in N$ ,  $x_i, y_i, z_i \in [0, 1]$  and  $\mathbf{w} \in \mathcal{X}$ , we have  $(x_i, w_{-i}) > (z_i, w_{-i})$  and  $(y_i, w_{-i}) > (z_i, w_{-i})$  implies that for all  $a \in [0, 1]$ ,  $(ax_i + (1 - a)y_i, w_{-i}) > (z_i, w_{-i})$ .

## 4. Main result

Our main result is summarized in the following theorem:

**Theorem 1.** A social welfare criterion  $\succeq$  satisfies A, S, SJ, NE, PUC and PSC, if and only if  $\succeq = \succeq^{SL}$ .

**Proof. If:**

It is obvious that  $\succeq^{SL}$  is a sufficientarian criterion as defined in Chambers and Ye (2024). Therefore, by their Theorem 1, it satisfies A (equivalent to Symmetry in their setting), S and SJ. Moreover, since  $[\underline{u}, \bar{u}] \subseteq ]0, 1[$ , it is easy to see that  $\succeq^{SL}$  satisfies NE. We have to show that it satisfies PUC and PSC.

To show PUC, take  $x_i \in [0, 1]$  and consider the following cases:

- **Case 1:**  $x_i \in [0, \underline{u} \cup ]\bar{u}, 1]$ . Then,  $\{y_i \in [0, 1] \mid \text{there exists } z_{-i} \in [0, 1]^{n-1} \text{ such that } (y_i, z_{-i}) \succeq^{SL} (x_i, z_{-i})\} = [0, 1]$ , which is closed.
- **Case 2:**  $x_i \in [\underline{u}, \bar{u}]$ . Then,  $\{y_i \in [0, 1] \mid \text{there exists } z_{-i} \in [0, 1]^{n-1} \text{ such that } (y_i, z_{-i}) \succeq^{SL} (x_i, z_{-i})\} = [\underline{u}, \bar{u}]$ , which is closed.

As the set  $\{y_i \in [0, 1] \mid \text{there exists } z_{-i} \in [0, 1]^{n-1} \text{ such that } (y_i, z_{-i}) \succeq^{SL} (x_i, z_{-i})\}$  is closed for any choice of  $x_i$ , this proves PUC.

We now show PSC. Let  $(x_i, w_{-i}) \succ^{SL} (z_i, w_{-i})$  and  $(y_i, w_{-i}) \succ^{SL} (z_i, w_{-i})$ , for some  $w \in \mathcal{X}$ ,  $i \in N$  and  $x_i, y_i, z_i \in [0, 1]$ . Then, clearly,  $x_i, y_i \in [\underline{u}, \bar{u}]$  and  $z_i \notin [\underline{u}, \bar{u}]$ . Thus, for all  $a \in [0, 1]$ , we have  $(ax_i + (1-a)y_i, w_{-i}) \succ^{SL} (z_i, w_{-i})$ . This completes the if part.

**Only if:**

Suppose that a social welfare criterion  $\succeq$  satisfies A, S, SJ, NE, PUC and PSC. First, we define a relation  $\succeq$  on  $[0, 1]$  such that, for all  $x, y \in [0, 1]$ ,  $x \succeq y \iff \text{there exists } i \in N \text{ and } z_{-i} \in [0, 1]^{n-1} \text{ such that } (x, z_{-i}) \succeq (y, z_{-i})$ .

Let the asymmetric part of  $\succeq$  be denoted by  $\triangleright$ . Note that, if for some  $i \in N$ ,  $x, y \in [0, 1]$  and  $z_{-i} \in [0, 1]^{n-1}$  we have  $(x, z_{-i}) \succeq (y, z_{-i})$ , then, by A, this holds for all  $i \in N$  and, moreover, by S, this holds for all  $z_{-i} \in [0, 1]^{n-1}$ . With these two observations, it is easy to show that  $\succeq$  is a weak order (complete and transitive).

We now claim that  $\succeq$  has two indifference classes. First, by NE, there exists  $\bar{x} = (\bar{x}_1, \dots, \bar{x}_n)$  such that, for all  $i \in N$ ,  $\bar{x} \triangleright (0, x_{-i})$ . Then,  $\bar{x} \triangleright 0$  and we establish that there are at least two indifference classes. We will now show that there are no more than two. Assume that this is not the case and take  $x, y, z \in [0, 1]$  such that  $x \triangleright y \triangleright z$ . Now, since  $y \triangleright z$ , we have  $(y, (y)_{-i}) \triangleright (z, (y)_{-i})$ . Then, by SJ we have  $(z, (y)_{-i}) \succeq ((x_i), (z_j), (y)_{-ij})$ . By A, we have  $((x_i), (z_j), (y)_{-ij}) \sim ((z_i), (x_j), (y)_{-ij})$  and thus  $((z_i), (y)_{-i}) \succeq ((z_i), (x_j), (y)_{-ij})$ . Finally, by S we get  $y \triangleright x$ , a contradiction of our previous assumption. So,  $\succeq$  has two indifference classes.

Let the highest of the two indifference classes be  $H$ . We now need to show that  $H$  is a closed interval. The fact that it is an interval follows directly from PSC. The fact that it is closed follows from PUC. Let then  $H = [\underline{u}, \bar{u}]$ . From NE, there exists  $\bar{x} = (\bar{x}_1, \dots, \bar{x}_n)$  such that  $\bar{x} \triangleright (0, \bar{x}_{-i})$  and  $\bar{x} \triangleright (1, \bar{x}_{-i})$ . It follows that  $\bar{x} \triangleright 0$  and  $\bar{x} \triangleright 1$  and thus  $\bar{x} \in [\underline{u}, \bar{u}]$ . This implies that  $\underline{u} \neq 0$  and  $\bar{u} \neq 1$  and hence,  $[\underline{u}, \bar{u}] \subseteq ]0, 1[$ , as required.

We finally show that, for all  $x, y \in \mathcal{X}$ ,

- (i)  $x \sim y \iff \#\{i \in N \mid x_i \in [\underline{u}, \bar{u}]\} = \#\{i \in N \mid y_i \in [\underline{u}, \bar{u}]\}$ , and
- (ii)  $x \triangleright y \iff \#\{i \in N \mid x_i \in [\underline{u}, \bar{u}]\} > \#\{i \in N \mid y_i \in [\underline{u}, \bar{u}]\}$

We first show (i). First, for any  $x \in \mathcal{X}$ , let  $K(x) = \{i \in N \mid x_i \in [\underline{u}, \bar{u}]\}$ . Now, assume that  $\#\{i \in N \mid x_i \in [\underline{u}, \bar{u}]\} = \#\{i \in N \mid y_i \in [\underline{u}, \bar{u}]\} = l \neq 0$  for some  $x, y \in \mathcal{X}$  and let  $K(x) = \{k_1^x, \dots, k_l^x\}$  and  $K(y) = \{k_1^y, \dots, k_l^y\}$ . Take permutations of  $x$  and  $y$ ,  $\bar{x}$  and  $\bar{y}$ , such that, for all  $i = 1, \dots, l$ ,  $\bar{x}_i = x_{k_i^x}$  and  $\bar{y}_i = y_{k_i^y}$ . Clearly, by A,  $\bar{x} \sim x$  and  $\bar{y} \sim y$ . Moreover, we have that, for all  $i \in N$ ,  $\bar{x}_i$  is in the same indifference class as  $\bar{y}_i$  with respect to  $\succeq$ . So, starting from  $\bar{x}_1$  and  $\bar{y}_1$ , we have  $\bar{x} \sim (\bar{x}_1, \dots, \bar{x}_n) \sim (\bar{y}_1, \bar{x}_2, \dots, \bar{x}_n)$ . By S,  $(\bar{y}_1, \bar{x}_2, \dots, \bar{x}_n) \sim (\bar{y}_1, \bar{y}_2, \bar{x}_3, \dots, \bar{x}_n) \sim (\bar{y}_1, \bar{y}_2, \bar{y}_3, \bar{x}_4, \dots, \bar{x}_n) \sim \dots \sim (\bar{y}_1, \dots, \bar{y}_n) = \bar{y}$ . Finally, since  $\bar{x} \sim x$  and  $\bar{y} \sim y$ , by transitivity, we get  $x \sim y$ . A similar argument holds if  $\#\{i \in N \mid x_i \in [\underline{u}, \bar{u}]\} = \#\{i \in N \mid y_i \in [\underline{u}, \bar{u}]\} = 0$ . In this case, for all  $i \in N$ ,  $x_i$  and  $y_i$  are in the same indifference class with respect to  $\succeq$ . Take  $x_1$  and  $y_1$ . We have then that  $x = (x_1, x_2, \dots, x_n) \sim (y_1, x_2, \dots, x_n)$  and, by S we have  $(y_1, x_2, \dots, x_n) \sim (y_1, y_2, x_3, \dots, x_n) \sim \dots \sim (y_1, y_2, \dots, y_n) = y$ , and thus  $x \sim y$ .

We now show (ii). Assume that  $\#\{i \in N \mid x_i \in [\underline{u}, \bar{u}]\} = m > l = \#\{i \in N \mid y_i \in [\underline{u}, \bar{u}]\}$  for some  $x, y \in \mathcal{X}$  and let  $K(x) = \{k_1^x, \dots, k_m^x\}$  and  $K(y) = \{k_1^y, \dots, k_l^y\}$ . Again, take permutations of  $x$  and  $y$ ,  $\bar{x}$  and  $\bar{y}$ , such that, for all  $i = 1, \dots, m$ ,  $\bar{x}_i = x_{k_i^x}$  and all  $j = 1, \dots, l$ ,  $\bar{y}_j = y_{k_j^y}$ . Then, note the following:

- For all  $i = 1, \dots, l$ ,  $\bar{x}_i$  and  $\bar{y}_i$  are in the same indifference class. So, by S, we have  $(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n) \sim (\bar{y}_1, \bar{x}_2, \dots, \bar{x}_n) \sim \dots \sim (\bar{y}_1, \dots, \bar{y}_l, \bar{x}_{l+1}, \dots, \bar{x}_n)$ .
- For all  $i = l + 1, \dots, m$ , we have  $\bar{x}_i \triangleright \bar{y}_i$ , so, by S, we have  $(\bar{y}_1, \dots, \bar{y}_l, \bar{x}_{l+1}, \dots, \bar{x}_n) \triangleright (\bar{y}_1, \dots, \bar{y}_l, \bar{y}_{l+1}, \bar{x}_{l+2}, \dots, \bar{x}_n) \triangleright \dots \triangleright (\bar{y}_1, \dots, \bar{y}_{m-1}, \bar{x}_m, \dots, \bar{x}_n)$ .
- For all  $i = m + 1, \dots, n$ ,  $\bar{x}_i$  and  $\bar{y}_i$  are in the same indifference class so, again, by S, we have  $(\bar{y}_1, \dots, \bar{y}_{m-1}, \bar{x}_m, \dots, \bar{x}_n) \sim (\bar{y}_1, \dots, \bar{y}_{m-1}, \bar{y}_m, \bar{x}_{m+1}, \dots, \bar{x}_n) \sim \dots \sim (\bar{y}_1, \dots, \bar{y}_n)$ .

Thus, by transitivity, we get  $x \sim (\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n) \sim (\bar{y}_1, \dots, \bar{y}_l, \bar{x}_{l+1}, \dots, \bar{x}_n) \triangleright (\bar{y}_1, \dots, \bar{y}_{m-1}, \bar{x}_m, \dots, \bar{x}_n) \sim (\bar{y}_1, \dots, \bar{y}_n) \sim y$  and  $x \triangleright y$  as required. This completes the proof.  $\square$

Our conditions are independent of each other. Below, we present six social welfare criteria, each of which satisfies all but one of our conditions:<sup>3</sup>

- (i) A social welfare criterion that satisfies all conditions but A is the following. Let  $g : N \rightarrow \mathbb{R}_{++}$ ,  $[\underline{u}, \bar{u}] \subseteq ]0, 1[$  and, for any  $x \in \mathcal{X}$ , let  $K(x)$  be defined as in the proof of our main theorem. Then, for any  $x, y \in \mathcal{X}$ ,  $x \succeq^g y \iff \sum_{i \in K(x)} g(i) \geq \sum_{i \in K(y)} g(i)$ . This criterion considers  $x$  to be better than  $y$  if the sum of the individual-specific weights of all individuals whose bundles lie between the sufficientarian and limitarian threshold, is greater in  $x$  than in  $y$ . The intuition is that, since the weight function  $g$  is agent specific,  $\succeq^g$  violates A. It is easy to verify that it satisfies all our other conditions.

- (ii) A social welfare criterion that satisfies all conditions but S is the following. For  $|N| \geq 3$  and all  $x, y \in \mathcal{X}$ ,

$$\begin{cases} \min[x] > \min[y] & \Rightarrow x \succ^m y, \text{ and} \\ \min[x] = \min[y] & \Rightarrow [x \succeq^m y \iff \max[x] \leq \max[y]]. \end{cases}$$

where  $\min[x]$  and  $\max[x]$  are the minimum and maximum elements of  $x$ , respectively. This lexicographic criterion first applies a maximin evaluation and, if the minimum is the same, then applies a minimax evaluation. This criterion naturally violates S, as the minimum and maximum elements are relative to the rest of the allocation. For example, we have  $(0.2, 0.3, 0.5) \succeq^m (0.1, 0.3, 0.5)$ , but  $(0.2, 0.1, 0.1) \prec^m (0.1, 0.1, 0.1)$ .

- (iii) A social welfare criterion that satisfies all conditions but SJ is the following. There exists  $[\underline{u}, \bar{u}] \subseteq ]0, 1[$  such that, for all  $x, y \in \mathcal{X}$ ,

$$\begin{cases} \#\{i \in N \mid x_i \geq \underline{u}\} > \#\{i \in N \mid y_i \geq \underline{u}\} & \Rightarrow x \succ_{lex}^{SL} y, \text{ and} \\ \#\{i \in N \mid x_i \geq \underline{u}\} = \#\{i \in N \mid y_i \geq \underline{u}\} & \Rightarrow [x \succeq_{lex}^{SL} y \iff \\ \#\{i \in N \mid x_i \leq \bar{u}\} \geq \#\{i \in N \mid y_i \leq \bar{u}\}]. \end{cases}$$

This is a lexicographic version of our sufficientarian–limitarian criterion. It first considers the number of individuals above the sufficientarian threshold and, if this number is the same in both allocations, then it considers the number of individuals below the limitarian threshold. To see why  $\succeq_{lex}^{SL}$  violates SJ, take  $[\underline{u}, \bar{u}] = [0.3, 0.7]$  and let  $x = (0.8, \dots, 0.8)$ . Then, notice that  $(0.8, \dots, 0.8) \succ_{lex}^{SL} (0.2, 0.8, \dots, 0.8)$ , but  $(0.2, 0.5, 0.8, \dots, 0.8) \succ_{lex}^{SL} (0.2, 0.8, \dots, 0.8)$ .

- (iv) A social welfare criterion that satisfies all conditions but NE is the trivial one, where for all  $x, y \in \mathcal{X}$ ,  $x \sim y$ .

- (v) A social welfare criterion that satisfies all conditions but PUC is the following. There exists  $[\underline{u}, \bar{u}] \subseteq ]0, 1[$  such that for all  $x, y \in \mathcal{X}$ :  $x \succeq_o^{SL} y \iff \#\{i \in N \mid x_i \in [\underline{u}, \bar{u}]\} \geq \#\{i \in N \mid y_i \in [\underline{u}, \bar{u}]\}$ . This is identical to our original criterion, except that the interval defined by the sufficientarian and limitarian thresholds is open. This naturally violates PUC but satisfies all other conditions.

<sup>3</sup> The formal proofs are available upon request.

- (vi) A social welfare criterion that satisfies all conditions but PSC is the following. There exist  $[u, \bar{u}], [\underline{v}, \bar{v}] \subseteq ]0, 1[$  with  $\underline{u} > \bar{v}$  such that, for all  $\mathbf{x}, \mathbf{y} \in \mathcal{X}$ :  $\mathbf{x} \succ^c \mathbf{y} \iff \#\{i \in N | x_i \in [\underline{u}, \bar{u}] \cup [\underline{v}, \bar{v}]\} \geq \#\{i \in N | y_i \in [\underline{u}, \bar{u}] \cup [\underline{v}, \bar{v}]\}$ . In this case, the set defined by the sufficientarian and limitarian thresholds consists of two non-overlapping closed intervals. Since this set is not convex, this criterion violates PSC.

## 5. Concluding remarks

In this paper, we characterized a social welfare criterion that integrates sufficientarian and limitarian principles. This criterion establishes the existence of a lower sufficientarian threshold and an upper limitarian threshold, such that for any two allocations  $\mathbf{x}$  and  $\mathbf{y}$ ,  $\mathbf{x}$  is considered better than  $\mathbf{y}$  if and only if the number of individuals within these thresholds is greater in  $\mathbf{x}$  than in  $\mathbf{y}$ . We demonstrate that a single normatively appealing axiom, *No Extremes*, distinguishes this hybrid criterion from headcount sufficientarianism.

These are several areas for further development of a sufficientarian–limitarian framework. One promising direction could involve incorporating prioritarian considerations below the sufficientarian threshold and above the limitarian threshold. This extension could address some criticisms directed to headcount sufficientarianism that are also relevant to sufficientarianism–limitarianism [e.g., it could recognize a change from a resource allocation (1,1) to (0.8,0.8) with a limit of 0.7 as a social improvement]. Another avenue for future research could explore the trade-offs between achieving sufficiency and avoiding excessive wealth by introducing different weights. For instance, a criterion could reflect the judgment that bringing someone above sufficiency is socially desirable even if it results in someone else exceeding the upper limit by a small amount. These refinements could deepen our understanding of the interplay between sufficientarian and limitarian principles. We leave this work for future research.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Acknowledgments

We are grateful to an anonymous referee, Susumu Cato, Chris Chambers, Takashi Hayashi, Marco Mariotti, and Paolo G. Piaquadio for very useful comments and discussions.

## Data availability

No data was used for the research described in the article.

## References

- Adler, M.D., Bossert, W., Cato, S., Kamaga, K., 2023. Ex-post approaches to prioritarianism and sufficientarianism. *Duke Law School Public Law & Legal Theory Series No.* 2023-53.
- Alcantud, J.C.R., Mariotti, M., Veneziani, R., 2022. Sufficientarianism. *Theor. Econ.* 17 (4), 1529–1557.
- Bloomberg, 2022. Macron backs pay cap for “CEOs” in bid for votes from the left. Available at: <https://www.bnnbloomberg.ca/macron-backs-pay-cap-for-ceos-in-bid-for-votes-from-the-left-1.1752688>.
- Bossert, W., Cato, S., Kamaga, K., 2022. Critical-level sufficientarianism. *J. Political Philos.* 30 (4), 434–461.
- Bossert, W., Cato, S., Kamaga, K., 2023. Thresholds, critical levels, and generalized sufficientarian principles. *Econom. Theory* 75 (4), 1099–1139.
- Casal, P., 2007. Why sufficiency is not enough. *Ethics* 117 (2), 296–326.
- Chambers, C.P., Ye, S., 2024. Haves and have-nots: A theory of economic sufficientarianism. *J. Econom. Theory* 217 (105805), 1–11.
- CNN, 2019. Democrats debate whether billionaires should exist. Available at: <https://www.cnn.com/2019/10/16/politics/wealth-tax-billionaires-elizabeth-warren-bernie-sanders/index.html>.
- Ferreira, J.V., Savva, F., Ramoglou, S., Vlassopoulos, M., 2024a. “Should CEOs’ salaries be capped?”: A survey experiment on limitarian preferences. *IZA Discussion Papers No.* 17171.
- Ferreira, J.V., Savva, F., Vlassopoulos, M., 2024b. Billionaire bankroll US politics, but voters could demand a fairer system. Available at: <https://theconversation.com/billionaires-bankroll-us-politics-but-voters-could-demand-a-fairer-system-245512>.
- Frankfurt, H., 1987. Equality as a moral ideal. *Ethics* 98 (1), 21–43.
- Frankfurt, H.G., 2015. *On inequality*. Princeton University Press.
- Gravel, N., Magdalou, B., Moyes, P., 2019. Inequality measurement with an ordinal and continuous variable. *Soc. Choice Welf.* 52, 453–475.
- Huseby, R., 2019. Sufficientarianism. In: *Oxford Research Encyclopedia of Politics*. Oxford University Press.
- Huseby, R., 2022. The limits of limitarianism. *J. Political Philos.* 30 (2), 230–248.
- Nakada, S., Sakamoto, N., 2024. The multi-threshold generalized sufficientarianism and level-oligarchy. Available at SSRN: <https://ssrn.com/abstract=4753486>.
- Nicklas, T., 2021. Rejecting ingrid robeyns’ defense of limitarianism. *Penn J. Philos. Politics & Econ.* 16 (1), 45–53.
- Perez-Truglia, R., Yusof, J., 2024. Billionaire superstar: Public image and demand for taxation. *National Bureau of Economic Research Working Paper Series No.* 32712.
- Robeyns, I., 2017. Having too much. *Nomos* 58, 1–44.
- Robeyns, I., 2022. Why limitarianism. *J. Political Philos.* 1–22.
- Robeyns, I., 2024. Limitarianism: The Case Against Extreme Wealth. *Random House*.
- Roemer, J.E., 2004. Eclectic distributional ethics. *Politics, Philos. Econ.* 3 (3), 267–281.
- Strain, M.R., 2024. In defense of billionaires. Available at: <https://www.project-syndicate.org/commentary/world-needs-more-billionaires-by-michael-r-strain-2024-01>.
- The Guardian, 2024. Limitarianism: why we need to put a cap on the super-rich. Available at: <https://www.theguardian.com/books/2024/jan/21/limitarianism-the-case-against-extreme-wealth-ingrid-robeyns-extract>.
- Timmer, D., 2021. Limitarianism: Pattern, principle, or presumption? *J. Appl. Philos.* 38 (5), 760–773.
- Zwarthoed, D., 2018. Autonomy-based reasons for limitarianism. *Ethical Theory Moral Pr.* 21 (5), 1181–1204.